

Lie-type transformations and effective Hamiltonians in nonlinear quantum optics: applications to multilevel systems

A. B. Klimov, A. Navarro

*Departamento de Física, Universidad de Guadalajara, Revolución 1500,
44420 Guadalajara, Jalisco, Mexico*

L. L. Sánchez-Soto

*Departamento de Óptica, Facultad de Ciencias Físicas, Universidad Complutense,
28040 Madrid, Spain*

Abstract

We reelaborate on a general method for diagonalizing a wide class of nonlinear Hamiltonians describing different quantum optical models. This method makes use of a nonlinear deformation of the usual $\text{su}(2)$ algebra and when some physical parameter, dictated by the particular model under consideration, becomes small, it gives a diagonal effective Hamiltonian that describes correctly the dynamics for arbitrary states and long times. We extend the technique to N -level atomic systems interacting with quantum fields, finding the corresponding effective Hamiltonians when the condition of k -photon resonance is fulfilled.

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1 Introduction

Many nonlinear quantum optical processes, such as k -th harmonic generation, k -wave mixing or generalized Dicke models, can be described in terms of operators from a nonlinear or deformed $\text{su}(2)$ algebra [1–8]. The emergence of this structure allows us to handle the problem in a close analogy with the usual treatment for an angular momentum: in consequence, we get a decomposition of the Hilbert space into direct sums of invariant subspaces and, therefore, the dynamical problem generated by the corresponding Hamiltonian can be reduced to the diagonalization of a finite-dimensional matrix. In spite of this considerable achievement, convenient analytic expressions for the eigenvalues

and eigenstates are unknown, although several approximations have been devised relying on the specific features of the particular problem under study.

Recently [9], the method of Lie transformations (see, e.g. Ref. 10 and references therein) has been applied to the approximate diagonalization of nonlinear optical Hamiltonians that can be recast in terms of the generators of polynomial deformed algebras. In this paper we further pursue this topic by applying the method to the relevant case of multilevel atomic systems interacting with single-mode quantized fields. Consequently, we find the corresponding effective Hamiltonians when the condition of k -photon resonance is fulfilled.

The contents of this paper are organized as follows: after a brief outline of the general method of small rotations developed in Ref. [9], we show how it works for multilevel systems and, finally, we give explicit examples for the cases of two- and three-photon resonances.

2 Nonlinear su(2) dynamics and effective Hamiltonians

Let us assume that the interaction Hamiltonian of the model under consideration can be written as

$$H_{\text{int}} = \Delta X_3 + gV, \quad (1)$$

where

$$V = X_+ + X_-. \quad (2)$$

Here g is a coupling constant that can be always chosen as real, Δ is a parameter usually representing the detuning between frequencies of different subsystems (although it is not necessary), and the operators X_{\pm} and X_3 maintain the first commutation relation of the su(2) algebra

$$[X_3, X_{\pm}] = \pm X_{\pm}, \quad (3)$$

while the second one is modified in the following way

$$[X_+, X_-] = P(X_3), \quad (4)$$

where $P(X_3)$ is an arbitrary polynomial function of the diagonal operator X_3 with coefficients that may depend on some integrals of motion N_j .

Now suppose that for some physical reason (depending on the particular model under consideration) the condition

$$\varepsilon = \frac{g}{\Delta} \ll 1 \quad (5)$$

is fulfilled. Then, it is clear that the Hamiltonian (1) is *almost* diagonal in the basis of the eigenstates of X_3 . In fact, a standard perturbation analysis immediately shows that the first-order corrections introduced by the nondiagonal part gV to the eigenvalues of X_3 vanish and those of second order are proportional to $g/\Delta \ll 1$. To proceed further we apply to (1) the following unitary transformation (which, in fact, is a *small* nonlinear rotation)

$$U = \exp(\varepsilon T), \quad (6)$$

with

$$T = X_+ - X_-, \quad (7)$$

in such a way that

$$H_{\text{eff}} = U H_{\text{int}} U^\dagger. \quad (8)$$

Using the standard expansion

$$e^A B e^{-A} = B + [A, B] + \frac{1}{2!} [A, [A, B]] + \dots, \quad (9)$$

we obtain

$$H_{\text{eff}} = \Delta X_3 + g \sum_{k=1}^{\infty} \varepsilon^k \frac{k}{(k+1)!} \text{ad}_T^k(V), \quad (10)$$

where ad_T is the adjoint operator defined as $\text{ad}_T(V) = [T, V]$. In other words, the effective Hamiltonian takes the form

$$\begin{aligned} H_{\text{eff}} = & \Delta X_3 - \varepsilon g \nabla \Phi(X_3) + \varepsilon^2 \frac{2g}{3} [X_+ \nabla^2 \Phi(X_3) + \nabla^2 \Phi(X_3) X_-] \\ & - \varepsilon^3 \frac{g}{4} \{X_+^2 \nabla^3 \Phi(X_3) + \nabla^3 \Phi(X_3) X_-^2 + 2 \nabla [\Phi(X_3) \nabla^2 \Phi(X_3 - 1)]\} \\ & + \mathcal{O}(\varepsilon^4). \end{aligned} \quad (11)$$

where $\Phi(X_3) = X_+ X_-$ is a structural function such that

$$P(X_3) = -\nabla \Phi(X_3) = \Phi(X_3) - \Phi(X_3 + 1). \quad (12)$$

By keeping terms up to order ε we finally get

$$H_{\text{eff}} = \Delta X_3 + \frac{g^2}{\Delta} P(X_3), \quad (13)$$

and the resulting effective Hamiltonian is diagonal in the basis of eigenstates of X_3 .

Furthermore, the higher-order contributions have always the form $X_+^k f(X_3) + f(X_3) X_-^k + g(X_3)$. This makes the procedure of removing the off-diagonal terms somehow trivial at each step, in the sense that it is always obvious the appropriate transformation that should be applied. For example, to eliminate terms of the form

$$\varepsilon^\ell [X_+^k f(X_3) + f(X_3) X_-^k], \quad (14)$$

it suffices with applying the transformation

$$\exp(\varepsilon^{\ell+1} T_k / k), \quad (15)$$

with $T_k = X_+^k f(X_3) - f(X_3) X_-^k$, since the first commutator of ΔX_3 with T_k cancels the corresponding term in the Hamiltonian.

With this approach, the evolution (as well as the spectral) problem is completely solved. The method has the virtue of generality, since it is valid for any model whose Hamiltonian could be written down in terms of the generators of an arbitrary polynomial deformation of $\text{su}(2)$, such as, for example, three-wave mixing, Dicke model or k -th harmonic generation [9]. Because the transformed Hamiltonian is an expansion in the small parameter ε , it is clear that its eigenvalues coincide with those obtained using the standard perturbation theory. The advantage of our method lies in obtaining *diagonal effective Hamiltonians* in an operator form, which avoids the necessity of calculating cumbersome series of matrix elements.

This technique also provides corrections to the eigenstates of the Hamiltonian (1). Indeed, from (8) it is easy to realize that the eigenstates of the interaction Hamiltonian (1) can be approximated as

$$|\Psi_m\rangle = U^\dagger |m\rangle, \quad (16)$$

where $|m\rangle$ is an eigenstate of X_3 and U is the corresponding small rotation. Since the rotation operators and $|m\rangle$ are time independent, the operator U can be applied to $|m\rangle$ in the form of an expansion in ε . For example, the eigenstate $|\Psi_m\rangle$ up to order ε^2 takes the form

$$\begin{aligned}
|\Psi_m\rangle &= |m\rangle - \varepsilon(X_+ - X_-)|m\rangle \\
&+ \frac{\varepsilon^2}{2}\{(X_+^2 + X_-^2) - [\Phi(X_3) + \Phi(X_3 + 1)]\}|m\rangle.
\end{aligned} \tag{17}$$

This representation is especially advantageous if we construct the space of states of the model as a representation space of the deformed $\text{su}(2)$ algebra using the raising operator X_+ [1–3]:

$$|m\rangle \propto X_+^m |0\rangle, \tag{18}$$

where $|0\rangle$ is the lowest weight vector fulfilling the standard condition $X_-|0\rangle = 0$.

3 Nonlinear $\text{su}(N)$ dynamics and effective Hamiltonians

The method of approximate diagonalization embodied in Eqs. (1), (6), and (13) can be applied also to systems with a more complicated algebraic structure.

In what follows we consider Hamiltonians which can be represented in terms of the $\text{su}(N)$ algebra. This algebraic structure naturally arises when describing atomic systems with N relevant energy levels.

The Hamiltonian governing the evolution of a collection of A identical N -level atoms (for definiteness, we consider the case of a cascade configuration, such that $E_i < E_j$ for $i < j$) interacting with a single-mode quantum field of frequency ω_f has the form (in units $\hbar = 1$)

$$H = \omega_f a^\dagger a + \sum_{j=1}^N \omega_j S^{jj} + \sum_{j=1}^{N-1} g_j (a S_+^{jj+1} + a^\dagger S_-^{jj+1}), \tag{19}$$

where S^{ii} ($i = 1, \dots, N$) are the population operators of the i -th level of energy ω_i , and $S_+^{ij} = S^{ij}$, $S_-^{ij} = (S_+^{ij})^\dagger$ ($i < j$) describe transitions between levels i and j . The operators S^{ij} form the algebra $\text{u}(N)$ and satisfy the commutation relations

$$[S^{ij}, S^{km}] = \delta_{im} S^{jk} + \delta_{jk} S^{im}. \tag{20}$$

Due to the integral of motion

$$\sum_{j=1}^N S^{jj} = A, \tag{21}$$

one can introduce inversion operators ($1 \leq j \leq N - 1$)

$$S_z^{jj+i} = \frac{1}{2}(S^{j+1j+1} - S^{jj}). \quad (22)$$

In this way the operators (S_{\pm}^{ij}, S_z^{jj+1}) form the algebra $\text{su}(N)$. Apart from (21), the Hamiltonian (19) possesses the following integral of motion

$$\hat{N} = a^{\dagger}a + \sum_{j=1}^{N-1} \mu_j S_z^{jj+1}, \quad (23)$$

with $\mu_j = j(N - j)$, that represent the total number of excitations in the system. Let us introduce detunings between energy levels of the atomic system according to

$$\Delta_j = \omega_j - \omega_1 - (j - 1)\omega_f, \quad (24)$$

with the trivial condition that $\Delta_1 = 0$. Moreover, let us assume that Δ_j satisfy the following resonant condition

$$\Delta_N = 0, \quad (25)$$

which means that the field is in a $(N - 1)$ -photon resonance with the atomic system: $\omega_N - \omega_1 = (N - 1)\omega_f$. Then, the Hamiltonian (19) can be rewritten as

$$H = H_0 + H_{\text{int}}, \quad (26)$$

with

$$H_0 = \omega_f \hat{N} + \omega A$$

$$H_{\text{int}} = h_0 + V, \quad (27)$$

where

$$\begin{aligned} \omega &= \frac{1}{2}(\omega_N + \omega_1), \\ h_0 &= \sum_{j=1}^N \Delta_j S^{jj}, \\ V &= \sum_{j=1}^{N-1} g_j (aS_+^{jj+1} + a^{\dagger}S_-^{jj+1}). \end{aligned} \quad (28)$$

One can observe that the operators ($1 \leq i < j \leq N$)

$$\begin{aligned} X_+^{ij} &= aS_+^{ij}, & X_-^{ij} &= a^\dagger S_-^{ij}, \\ X^{kk} &= S^{kk}, \end{aligned} \quad (29)$$

form a polynomial deformation of the algebra $\mathfrak{u}(N)$, since they satisfy the following commutation relations

$$\begin{aligned} [X^{ii}, X^{kk}] &= 0, & [X_+^{ij}, X_+^{ik}] &= 0, & [X_-^{ij}, X_-^{ik}] &= 0, \\ [X^{kk}, X_\pm^{ij}] &= \pm X_\pm^{ij}(\delta_{kj} - \delta_{ik}), \\ [X_+^{ij}, X_-^{ij}] &= P(X^{ii}, X^{kk}), \end{aligned} \quad (30)$$

where $P(X^{ii}, X^{kk})$ is a polynomial function of the diagonal operators X^{ii} and the integral of motion \hat{N} .

According to the general scheme (6)-(13) we introduce the following transformation

$$U_1 = \exp(T_1), \quad (31)$$

with

$$T_1 = \sum_{j=1}^{N-1} \alpha_j^{(1)} (aS_+^{jj+1} - a^\dagger S_-^{jj+1}), \quad (32)$$

where the parameters $\alpha_j^{(1)}$ are

$$\alpha_j^{(1)} = \frac{g_j}{\Delta_{j+1} - \Delta_j} \quad (33)$$

and are assumed to be small numbers, $\alpha_j^{(1)} \ll 1$, which means that the atomic transitions are far from the one-photon resonance ($\Delta_{j+1} - \Delta_j = \omega_{j+1} - \omega_j - \omega_f \gg g_j$). Since

$$[T_1, h_0] = -V, \quad (34)$$

all one-photon transitions are eliminated by (32) and the transformed Hamiltonian $H_{\text{eff}}^{(1)} = U_1 H_{\text{int}} U_1^\dagger$ takes the form

$$H_{\text{eff}}^{(1)} = h_0 + h_{\text{diag}} + h_{\text{nondiag}} + \sum_{k=1}^{N-2} \frac{k}{(k+1)!} \sum_{j=1}^{N-k-1} \psi_j^{(k+1)} (a^{k+1} S_+^{jj+k+1} + a^{\dagger k+1} S_-^{jj+k+1}), \quad (35)$$

where the effective interaction constants $\psi_j^{(k)}$ can be obtained from the recurrence relation

$$\psi_j^{(k+1)} = \alpha_{j+k}^{(1)} \psi_j^{(k)} - \alpha_j^{(1)} \psi_{j+1}^{(k)}, \quad (36)$$

with the initial term $\psi_j^{(1)} = g_j$. It is easy to see that $\psi_j^{(k)} \sim \alpha_j^{(k-1)}$ and thus $\psi_j^{(k+1)} \ll \psi_j^{(k)}$.

The term h_{diag} contains only diagonal terms in the atomic operators and depends on the integral of motion \hat{N} (or, which is equivalent, depends only on the photon-number operator $a^\dagger a$). This operator h_{diag} appears naturally represented as an expansion on the small parameter $\alpha_j^{(1)}$ whose first term has the following form

$$h_{\text{diag}} = \frac{1}{2} \sum_{j=1}^{N-1} g_j \alpha_j^{(1)} [S_z^{jj+1} (2a^\dagger a + 1) + \{S_+^{jj+1}, S_-^{jj+1}\}_+] + \mathcal{O}(\alpha^3). \quad (37)$$

The essential point is that this diagonal operator cannot be removed from the effective Hamiltonian (35). On the contrary, the operator h_{nondiag} contains only nondiagonal terms that can be eliminated by rotations of the type (15). In this respect, let us note that the price we must pay for eliminating one-photon transitions is the generation of all possible (in this atomic configuration) k -photon transitions ($k = 2, \dots, N-1$). The possibility of removing some of the terms in the sum appeared in (35) strongly depends on the resonance conditions. In the next section we shall examine this delicate point by means of some examples.

4 Examples

4.1 Two-photon resonance.

Let us consider the interaction of a collection of A identical three-level atoms ($N = 3$) in a cascade configuration with a single-mode quantum field of frequency ω_f and suppose that all the atomic transitions are far from the one-photon resonance with the field. Then, these transitions can be eliminated

using the transformation (32). Taking into account the two-photon resonance condition $\Delta_3 = 0$ (i.e.; $\omega_3 - \omega_1 = 2\omega_f$) and keeping terms up to order $\alpha_j^{(1)}$, we obtain from (35) an effective Hamiltonian for two-photon transitions (the nondiagonal part, h_{nondiag} , can be eliminated by an appropriate transformation and no contributions of order $\alpha_j^{(1)}$ appear)

$$H_{\text{eff}}^{(1)} = h_0 + h_{\text{diag}} + \frac{1}{2}\psi_1^{(2)}(a^2 S_+^{13} + a^{\dagger 2} S_-^{13}), \quad (38)$$

where

$$h_{\text{diag}} = \frac{1}{2} \sum_{j=1}^2 g_j \alpha_j^{(1)} [S_z^{jj+1} (2a^\dagger a + 1) + \{S_+^{jj+1}, S_-^{jj+1}\}_+], \quad (39)$$

and the interaction constants, from (33) and (36), result to be

$$\begin{aligned} \alpha_1^{(1)} &= \frac{g_1}{\Delta_2}, & \alpha_2^{(1)} &= -\frac{g_2}{\Delta_2}, \\ \psi_1^{(2)} &= \alpha_2^{(1)} g_1 - \alpha_1^{(1)} g_2 = -2 \frac{g_1 g_2}{\Delta_2}. \end{aligned} \quad (40)$$

Taking into account that $S^{11} + S^{22} + S^{33} = A$ and imposing the absence of initial population in level 2 (note that the term h_0 disappears in this case), we obtain the effective two-photon Hamiltonian including the dynamical Stark shift [11–13]

$$\begin{aligned} H_{\text{eff}} &= \frac{g_1 g_2}{\Delta_2} (a^2 S_+^{13} + a^{\dagger 2} S_-^{13}) \\ &+ (S_z^{13} + A/2) \left[\left(\frac{g_2^2 - g_1^2}{\Delta_2} \right) a^\dagger a + \frac{g_2^2}{\Delta_2} \right] + A \frac{g_1^2}{\Delta_2} a^\dagger a. \end{aligned} \quad (41)$$

4.2 Three-photon resonance

Let us consider the interaction of a collection of A identical four-level atoms ($N = 4$) in a cascade configuration with a single-mode quantum field of frequency ω_f and suppose that there are no one- and two-photon resonances with the field.

After eliminating one-photon transitions the transformed Hamiltonian (35) (written up to the corresponding order) has the form

$$H_{\text{eff}}^{(1)} = h_0 + h_{\text{diag}} + h_{\text{nondiag}} \\ + \frac{1}{2} \sum_{j=1}^2 \psi_j^{(2)} (a^2 S_+^{jj+2} + a^{\dagger 2} S_-^{jj+2}) + \frac{1}{3} \psi_1^{(3)} (a^3 S_+^{14} + a^{\dagger 3} S_-^{14}), \quad (42)$$

where

$$h_{\text{diag}} = \frac{1}{2} \sum_{j=1}^3 g_j \alpha_j^{(1)} [S_z^{jj+1} (2a^\dagger a + 1) + \{S_+^{jj+1}, S_-^{jj+1}\}_+] + \mathcal{O}(\alpha^3), \quad (43)$$

and the essential part of the nondiagonal operator h_{nondiag} is

$$h_{\text{nondiag}} = \frac{1}{2} \sum_{i,j=1}^3 \alpha_i^{(1)} g_j (S_+^{ii+1} S_-^{jj+1} + S_+^{jj+1} S_-^{ii+1}) + \mathcal{O}(\alpha^2). \quad (44)$$

The interaction constants are defined according to (33) and (36):

$$\alpha_1^{(1)} = \frac{g_1}{\Delta_2}, \quad \alpha_2^{(1)} = \frac{g_2}{\Delta_3 - \Delta_2}, \quad \alpha_3^{(1)} = -\frac{g_3}{\Delta_3}, \\ \psi_1^{(2)} = g_1 g_2 \frac{2\Delta_2 - \Delta_3}{\Delta_2(\Delta_3 - \Delta_2)}, \quad \psi_2^{(2)} = g_2 g_3 \frac{2\Delta_3 - \Delta_2}{\Delta_3(\Delta_2 - \Delta_3)}, \\ \psi_1^{(3)} = \frac{3g_1 g_2 g_3}{\Delta_3 \Delta_2}, \quad (45)$$

where, in addition, the resonance condition $\Delta_4 = 0$ (i.e., three-photon resonance $\omega_4 - \omega_1 = 3\omega_f$) has been imposed.

According to the general scheme, the second term (representing two-photon transitions) in (42) can be removed by the unitary transformation

$$U_2^{(1)} = \exp[T_2^{(1)}], \quad (46)$$

with

$$T_2^{(1)} = \frac{1}{2} \sum_{j=1}^2 \alpha_j^{(2)} (a^2 S_+^{jj+2} - a^{\dagger 2} S_-^{jj+2}), \quad (47)$$

where now

$$\alpha_j^{(2)} = \frac{\psi_j^2}{\Delta_{j+2} - \Delta_j} \quad (48)$$

are also assumed to be small parameters; i.e., there are no resonant two-photon transitions ($\Delta_{j+2} - \Delta_j = \omega_{j+2} - \omega_j - 2\omega_f \gg \psi_j^{(2)}$) and thus $\alpha_j^{(2)} \ll \alpha_j^{(1)}$.

The nondiagonal term h_{nondiag} can be also eliminated (in the same approximation) with the following transformation

$$U_2^{(2)} = \exp[T_2^{(2)}], \quad (49)$$

where

$$T_2^{(2)} = \frac{1}{2} \sum_{i,j=1}^3 \beta_{ij} (S_+^{ii+1} S_-^{jj+1} + S_+^{jj+1} S_-^{ii+1}), \quad (50)$$

and

$$\beta_{ij} = \frac{\alpha_i^{(1)} g_j}{\Delta_{i+1} - \Delta_i + \Delta_j - \Delta_{j+1}}. \quad (51)$$

Taking into account that $S^{11} + S^{22} + S^{33} + S^{44} = A$ and imposing the condition of the absence of initial population in levels 2 and 3, we finally obtain the effective Hamiltonian describing three-photon resonant transitions

$$\begin{aligned} H_{\text{eff}} = & \frac{1}{3} \psi_1^{(3)} (a^3 S_+^{14} + a^{\dagger 3} S_-^{14}) \\ & - \frac{1}{2} [\alpha_1^{(1)} g_1 (a^\dagger a - 1/2) S^{11} - \alpha_3^{(1)} g_3 (a^\dagger a + 3/2) S^{44}] \\ & - \frac{1}{2} \beta_{31} S^{11} S^{44} \\ & - \frac{1}{4} [\alpha_1^{(2)} \psi_1^{(2)} a^\dagger a (a^\dagger a - 1) S^{11} - \alpha_2^{(2)} \psi_2^{(2)} (a^\dagger a + 1) (a^\dagger a + 2) S^{44}]. \end{aligned} \quad (52)$$

One must note that the two last terms are of order $1/\Delta^3$ and, thus, can be omitted in the effective Hamiltonian, because the interaction term generating transitions between levels 1 and 4 is of order $1/\Delta^2$. Then, the effective Hamiltonian takes the form

$$\begin{aligned} H_{\text{eff}} = & \frac{3g_1 g_2 g_3}{\Delta_2 \Delta_3} (a^3 S_+^{14} + a^{\dagger 3} S_-^{14}) \\ & - \frac{1}{2} \left[\frac{g_1^2}{\Delta_2} (a^\dagger a - 1/2) (A/2 - S_z^{14}) + \frac{g_3^2}{\Delta_3} (a^\dagger a + 3/2) (A/2 + S_z^{14}) \right], \end{aligned} \quad (53)$$

where $S_z^{14} = (S^{44} - S^{11})/2$.

As in the two-photon case, the three-photon Hamiltonian (53) contains the dynamical Stark shift (with the corresponding spontaneous contribution [11]). Nevertheless, the essential difference between these two cases lies in the fact

that in the two-photon case the interaction term and the Stark-shift term are of the same order of magnitude, while in the three-photon case the interaction term is one order of magnitude lesser than the Stark-shift term. This lead to essential differences in the evolution of some observables. In the same way, k -photon effective interaction Hamiltonian can be obtained.

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